DEFINITE INTEGRATION

Properties of Definite Integration

Property 1

If, $\int_a^b f(x).dx = 0$. Then, the equation f(x) has at-least 1 root in (a, b), provided f(x) is continuous

Property 2

$$lt_{n\to\infty}\left(\int_a^b f(n,x).dx\right) = \int_a^b lt_{n\to\infty} f(n,x).dx$$

Property 3

$$\int_a^b \frac{d}{dx}(f(x)).dx = f(x)|_a^b = f(b) - f(a)$$

However, if f(x) is discontinuous in a,b at x = c

$$\int_a^b \frac{d}{dx}(f(x)).dx = \int_a^{c-} \frac{d}{dx}(f(x)).dx + \int_{c+}^b \frac{d}{dx}(f(x)).dx$$

Property 4

If g(x) is the inverse of f(x) and f(a) = c & f(b) = d

$$\int_a^b f(x).dx + \int_c^d g(x).dx = (bd - ac)$$

Property 5

$$\int_a^b f(x).dx = \int_a^b f(t).dt$$

x = t (Independent of choice of independent variable).

Property 6

$$\int_a^b f(x).dx = -\int_b^a f(x).dx$$

Property 7 (used for |x|, {x}, [x])

$$\int_a^b f(x).dx = \int_a^c f(x).dx + \int_c^b f(x).dx$$

Property 8

$$\int_{-a}^a f(x).dx = \int_0^a \big(f(x) + f(-x)\big).dx$$

0, if f(x) is odd	$2\int_0^a f(x).dx$ if f(x) is even

Property 9

$$\int_a^b f(x).dx = \int_a^b f(a+b-x).dx$$

Property 10

$$\int_a^{2a} f(x).\,dx = \int_0^a f(x).\,dx + \int_0^a f(2a-x).\,dx$$

$$= \begin{cases} 0, & \text{if } f(2a-x) = -f(x) \\ 2\int_0^2 f(x).\,dx, & \text{if } f(2a-x) = f(x) \end{cases}$$

Property 11

$$\int_a^b f(x).\,dx = (b-a)\int_0^1 f\big((b-a)x + a\big)dx$$

Newton-Leibnitz rule

$$\frac{d}{dx}\left(\int_{y_0}^{y_1} f(x,y)dy\right) = \int_{y_0}^{y_1} f_x(x,y)dy$$

DI of Periodic Functions

Property 1

$$\int_0^{nT} f(x).\,dx = n.\int_0^T f(x).\,dx$$

where 'T' is the period of the function and $n \in N$.

Property 2

$$\int_0^{a+nT} f(x).\,dx = n.\int_0^{T} f(x).\,dx$$

Property 3

$$\int_{mT}^{nT} f(x).\,dx = (n-m).\int_0^{T} f(x).\,dx$$

Property 4

$$\int_{a+nT}^{b+nT} f(x).\,dx = \int_a^{b} f(x).\,dx$$

Limit of the sum

$$\lim_{n\to\infty}\sum_{r=r_1}^{r=r_2}\frac{1}{n}.f\left(\frac{r}{n}\right) = \int_{\lim\limits_{n\to\infty}\frac{r_1}{n}}^{\lim\limits_{n\to\infty}\frac{r_2}{n}} dx.\,f(x)$$

Limit of the sum

$$\int_0^{\pi/2} \sin^n x dx = \int_0^{\pi/2} \cos^n x dx \qquad k = \begin{cases} \frac{\pi}{2}, & \text{if n is even} \\ 1, & \text{if n is odd} \end{cases}$$

$$= \frac{(n-1)(n-3)(n-3)\ldots(1 \text{ or } 2)}{n(n-2)(n-4)\ldots(1 \text{ or } 2)}.k$$